

Inertia and Double Bending of Light from Equivalence

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Careful examination of light paths in an accelerated reference frame, with use of Special Relativity, can account fully for the observed bending of light in a gravitational field, not just half of it as reported in 1911. This analysis also leads to a Machian formulation of inertia similar to the one proposed by Einstein in 1912 and later derived from gravitational field equations in Minkowsky Space by Sciama in 1953.

I. INTRODUCTION

Three remarkable ideas have competed with Newton's notion of absolute space and time for the last 100 years. Each of these is compelling, yet claims of incorporation of all three into a theory of gravity are controversial.

First is the idea of the relativity of inertia, sometimes called Mach's Principle since among philosophers who espoused it he is most often quoted. Mach felt that motion was only definable relative to other matter, and that water would crawl up the sides of a bucket if the universe rotated around it, instead of rotating the bucket [1]. While Mach did not quantify this theory, Einstein did in a 1912 paper [2] relating inertia to gravitational potential energy divided by c^2 to convert the energy to a mass term. In 1917, convinced that General Relativity Theory (GRT) incorporated Mach's Principle, Einstein said, "In a consistent theory of relativity there can be no inertia relatively to space, but only an inertia of masses relatively to one another. If, therefore, I remove a mass to a sufficient distance from all other masses in the universe, its inertia must fall to zero [3]." His 1921 book defends this position in detail [4], but by 1949 he has reversed himself and lamented, "...the attempt at such a solution does not fit into a consistent field theory..." [also 4] In 1962 Brans argued the only way GRT can influence matter is through the metric, which can be transformed away for an arbitrarily small laboratory [5].

Second is the Principle of Equivalence, primarily due to Einstein but rooted in observations from Galileo to Eötvös that all objects fall at the same rate. The first or "weak" form of equivalence reasons that therefore gravity must act equally on all forms of inertia (or energy). The "strong" form used in the formulation of

GRT holds that in the neighborhood of a point in space, all physical experiments will give the same result whether conducted in an accelerated frame of reference, or supported (as on a planetary surface) in a gravitational field. This gives a rationale for the effect of gravity on light, since light crossing an accelerated elevator (or rocket) would appear to bend to an observer in the elevator. In 1911 Einstein published a prediction of light bending near the sun based on this argument, without using Special Relativity [6]. Fortunately for him it wasn't tested, because it only gave half the correct value. Later the full GRT derived the correct value from the curvature of space-time, but equivalence was relegated to an incomplete role, valid only for infinitesimally small Local Inertial Frames (LIFs).

The third idea is that since gravity seems to affect all objects and energy, perhaps it is a property of space-time itself, a curvature of space-time. This elegant idea has led to useful theories of cosmology, and explanations of light paths and orbital precession, but seems to stand in the way of integration of GRT with inertia and also possibly with quantum theory.

In this paper I will show that equivalence gives a full account of light bending when Special Relativity is used, and leads to the same equation for inertia given by Einstein in 1912 and by Sciama in 1953 [7]. This implies any theory based on equivalence should accommodate the relation for inertia, and an equivalent sort of light bending.

II. LORENTZ POTENTIAL

First we will derive a Lorentz potential, analogous to a gravitational potential, based on velocity differences in

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an accelerated frame. This will imply relative differences in time, mass and motion in different parts of the frame. Since an accelerated frame is not inertial, we need not expect measurements to be uniform in all parts of it.

Consider an elevator either in free space undergoing acceleration a or suspended in a gravitational field of strength $g = a$. According to the equivalence principle, locally it doesn't matter which. Figure 1 shows the elevator as seen by an observer who is co-moving with the elevator at an initial time when light is sent horizontally from coordinates 0.5 and 1.0. The dashed outline shows the elevator at a later time when the light exits the elevator. The elevator not only has moved, but it appears Lorentz contracted since it is now moving relative to the observer. The effect is exaggerated for illustration, and the new length scale is shown on the right.

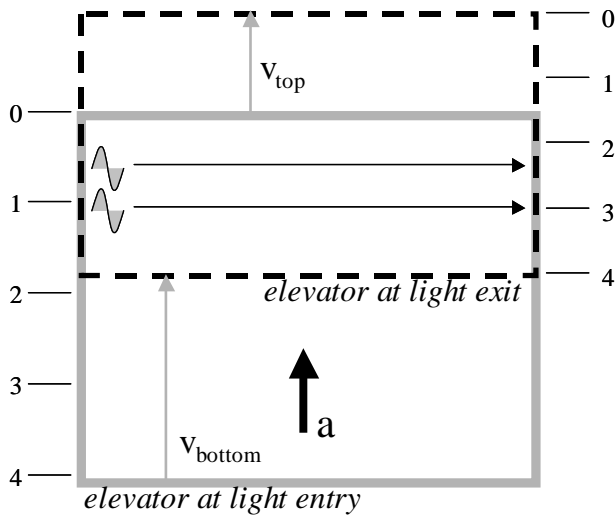


Fig. 1 Light Crossing Elevator
(initially co-moving inertial view)

From within the elevator, the light appears to follow a parabolic path, shown in Figure 2. Because of the contraction of the elevator, the exit points of the light are separated by a greater distance Δh than the entry points. The gray dashed line shows the path of the upper light ray displaced downward by Δh . It does not reach the actual exit point of the lower ray. Since the lower ray travels further, it takes more time to reach its exit. Another way of saying this is that it travels slower. Although its speed will always be “ c ” measured locally,

an observer Δh above the light perceives it moving slower. The inertial observer sees the light rays exiting at the same time, but his clocks are no longer synchronized with the elevator clocks, after the elevator has increased its speed.

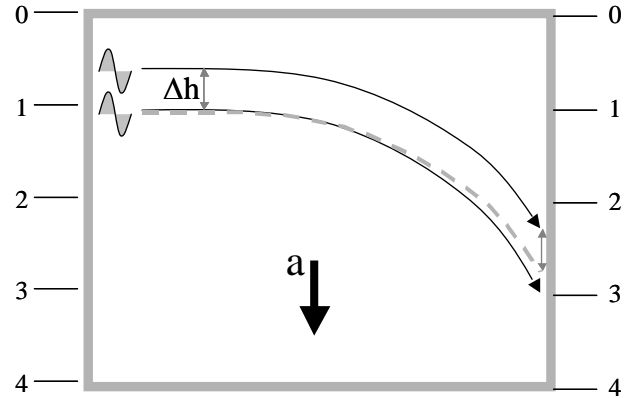


Fig. 2 Light Crossing Elevator
(view from elevator reference frame)

So there are two effects due to relativistic changes in the elevator, as viewed from an inertial frame. Lower objects fall further due to the progressive contraction of the elevator. This is only slightly greater than Newtonian falling. But objects, or light rays, that are moving horizontally also “turn” downward since their lower portions travel further (or in the elevator frame, slower) and do not reach the exit at the same time as their upper portions. This latter effect is the one we will use. We will calculate a “speed gradient refraction” of the light. This method will be useful also in other situations where the geometry is not obvious.

Speed gradient refraction is analogous to Huygens refraction, and results in deflections identical to Huygens refraction for light, but the speed gradient may be used for objects that are not primarily described by wave motion. In fact we will generalize our computation of the deflection due to the speed gradient to apply to any object moving with any speed across the elevator. Even if the object is a point particle, the probability function of its future occurrences will be shaped by the speed gradient.

In the inertial view, points lower in the elevator must be traveling faster (vertically), due to the progressive

contraction. To determine light speed slowing or time dilation or inertial mass increase, we must determine the Lorentz factor, $\gamma = 1/(1-v^2/c^2)^{0.5}$. We wish to know the γ observed from a first point in the elevator, for a second point at a distance Δh lower traveling Δv_h faster. Since Δv_h will be quite small, we will use approximations to derive

$$\gamma = 1 / \sqrt{1 - \frac{v^2}{c^2}} \approx 1 / (1 - .5 \frac{v^2}{c^2}) \approx 1 + \frac{1}{2} \frac{v^2}{c^2} \approx 1 + \frac{v}{c}$$

and will freely interchange $1+v/c$ with $1/(1-v/c)$.

For a co-moving inertial observer at the first (upper) point, the contracted interval after some Δt is $\Delta h' = \Delta h/\gamma$. The vertical velocity v_{12} of the second point with respect to the first is the change in Δh , divided by Δt . So we have $v_{12} = (\Delta h - \Delta h')/\Delta t = \Delta h(1-1/\gamma)/\Delta t \approx (\Delta h/\Delta t)(1-(1-v_h/c)) \approx (\Delta h/\Delta t)(v_h/c)$ where v_h is vertical velocity after interval Δt given by $a\Delta t$. Substituting for v_h we have $v_{12} = (\Delta h/\Delta t)(a\Delta t/c)$. The Δt 's cancel leaving $v_{12} = a\Delta h/c$. We can now compute the Lorentz potential (γ) of the second point with respect to the first:

$$\gamma_{12} \approx 1 + \frac{v_{12}}{c} = 1 + \frac{a\Delta h}{c^2} \quad (2.1)$$

If we have objects at three points spaced at Δh , such that the Lorentz properties of the 3rd are modified by γ_{23} from the point of view of the 2nd, and all properties viewed by the 2nd are modified by γ_{12} from the point of view of the first, then it follows that:

$$\gamma_{13} = \gamma_{12}\gamma_{23} = (1 + \frac{a\Delta h}{c^2})^2 = 1 + 2\frac{a\Delta h}{c^2} + (\frac{a\Delta h}{c^2})^2 \quad (2.2)$$

Equation (2.2) reduces to (2.1) when taking Δh to be the distance from 1 to 3, if the higher order term is small. It may be extended further with higher order polynomials.

III. DOUBLE BENDING OF LIGHT

Referring to Figure 3, consider two parts of a wave or particle separated by Δh and traveling horizontally at v and v_2 respectively.

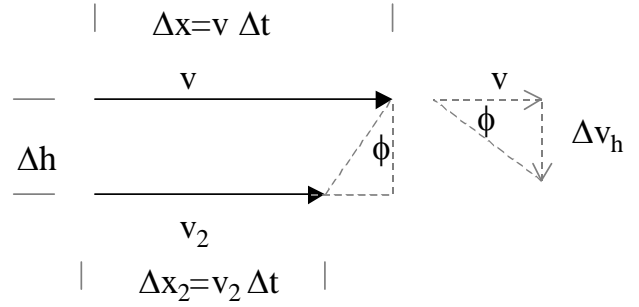


Fig. 3 Setup for Speed Gradient Refraction

After a horizontal interval Δx we have $\Delta x = v\Delta t$, and we assume $\Delta x_2 = v_2\Delta t = (v/\gamma)\Delta t$, (for $v = c$ this assumption must be valid, and we'll treat $v < c$ later). Two formerly vertical points on the object will be turned at an angle ϕ such that $\sin \phi \approx \phi \approx (\Delta x - \Delta x_2)/\Delta h = (v - v/\gamma)\Delta t/\Delta h$. The velocity vector v will be turned by this same angle ϕ so that a vertical velocity component Δv_h is added, where $\sin \phi \approx \phi \approx \Delta v_h/v$. Equating the two expressions for ϕ we have $\phi \approx \Delta v_h/v = (v - v/\gamma)\Delta t/\Delta h$. We can rearrange this into an expression for $\Delta v_h/\Delta t = v^2(1 - 1/\gamma)/\Delta h$. This value $\Delta v_h/\Delta t$ is aligned with the acceleration a . Substituting for γ using (2.1) and simplifying we have

$$\frac{\Delta v_h}{\Delta t} = v^2(1 - 1 + a\Delta h / c^2) / \Delta h = \frac{v^2}{c^2} a \quad (3.1)$$

For light, we have $v = c$ and therefore $\Delta v_h/\Delta t = a$. Since $\Delta v_h/\Delta t$ is added to the explicit acceleration, a , as already noted, we have a total apparent acceleration of $2a$. Thus the double bending of light is derived from the Principle of Equivalence and Special Relativity Theory (SRT).

Some investigators have been puzzled at the coincidence that Huygens refraction bending should be exactly equal to Newtonian bending. From the foregoing we conclude that whenever $v = c$ this will be the case.

Before asking whether equation (3.1) applies to objects traveling at other speeds, we need to learn more about inertia.

IV. INERTIA FROM EQUIVALENCE

Notice that $a\Delta h$ in (2.1) is gravitational potential, which becomes an inertial mass term when divided by c^2 .

Extending (2.2) in a non-uniform $1/R^2$ field between masses M and m , we assert that to first order:

$$\gamma = 1 + GM / Rc^2 \quad (4.1)$$

For this paper we simply note that the functional similarity of ah/c^2 and GM/Rc^2 justify (4.1). In a separate paper there will be more discussion and an alternate derivation of (4.1) from the basic principles of inertia. Here we are going backward from equivalence toward inertia.

Based on equivalence, and using either (2.1) or (4.1), we see that certain features of the Lorentz transform can be applied to a gravitational field. As before, we need to know the “delta” γ between two points (or reference frames), 1 and 2. Suppose these are at radii R_1 and R_2 . Equation (4.1) is based on the potential γ_1 or γ_2 of R_1 or R_2 with respect to infinite R . We can apply a relation like (2.2) but with unequal radii, and solve for the desired relative γ .

$$\gamma_{\infty 2} = \gamma_{\infty 1} \gamma_{12} \quad (4.2)$$

$$\Rightarrow \gamma_{12} = \frac{\gamma_{\infty 2}}{\gamma_{\infty 1}} = \frac{\gamma_2}{\gamma_1} = \frac{1 + GM / R_2 c^2}{1 + GM / R_1 c^2} \quad (4.3)$$

Using (4.3) we can calculate the γ we should apply from any point of view (frame) in a gravitational field to in-frame measurements at a 2nd point in the field, to convert them into measurements valid in the first point of view.

If in addition there are velocity differences, we will need to multiply by a Lorentz factor γ . To avoid confusion, or excessive subscripting, I will adopt the convention that the gravitational γ will be represented by Γ . So we have a total reference frame adjustment factor of $\Gamma\gamma$, where $\Gamma = 1 + GM/Rc^2$ and $\gamma = 1/(1 - v^2/c^2)^{0.5}$.

We can formulate a partial list of the laws of inertia, those implied by equivalence, where the primed quantities are the measurements taken from another reference frame, as follows:

$$T' = T / \Gamma\gamma \quad (4.4)$$

$$mi' = mi\Gamma\gamma \quad (4.5)$$

$$x' = x / \gamma \quad (4.6)$$

Equivalence and SRT provide no insight about gravitational mass, only the inertia of objects, therefore (4.5) references an increase only in inertial mass, here designated mi . Whether the traditional notion that the gravitational mass is also implied in (4.5) can be accommodated is somewhat an open question, but if total mass is a function of itself we will be presented with a difficult paradox.

Analysis of the accelerated reference frame shows time dilation and mass increase due to the higher velocity v_h of objects lower in the accelerated frame, when viewed from any inertial reference frame. To obtain consistent results in physical reality (i.e. events coincident in both time and space are coincident in all reference frames), observers in the accelerated frame must also see time dilation for objects lower in the frame, and apparent inertial mass increase of those objects. One can infer from mass increase and conservation of momentum that the objects must also slow down, or one can infer the slowing from time dilation, hence our earlier assumption $v_2 = (v/\gamma)$. Thus the formula (3.1) for speed gradient refraction applies to objects moving at any velocity.

Using equivalence, we can transfer these results to the gravitational field, where the time dilation and mass increase factor becomes the gravitational potential instead of Lorentz potential. If we examine Newton's equation for inertia, $F=ma$, or more precisely $F=(mi)a$ where mi is inertial mass, we see that if we write this equation for objects at a different location than our own in a gravitational field, we must use the relative $\Gamma\gamma$ factor between the two locations as defined in (4.3).

$$F = (mi')a = (mi\Gamma_{12}\gamma_{12})a \quad (4.7)$$

If we look at only the gravitational portion, and take a reference point at infinity, or in space where gravitational forces are balanced, then we have

$$F = (mi')a = mi(1 + GM / Rc^2)a \quad (4.8)$$

Clearly $1 + GM/Rc^2$ is an inertia adjustment factor based on gravitational potential. It has exactly the same form as Einstein's calculation of the inertia due to a spherical shell of mass at radius R in his 1912 paper [2].

If we follow Sciamia [7] and make allowances for notation and units (c.f. also Ghosh [8]), we find an equation for the force due to gravity having a $1/R^2$ term dependent only on position and the two masses and the gravitational constant (the Newtonian term), and a second term dependent on acceleration and $1/Rc^2$.

$$F = \frac{GmM}{R^2} + a \frac{GmM}{Rc^2} \quad (4.9)$$

Let m be a mass of interest, and $M_{k..n}$ be all the remaining mass objects in the visible universe. Then the total F (force due to gravity) will be a summation over all the $M_{k..n}$. In deep space far from any locally gravitating objects, we assume an isotropic universe so the first summation is approximately zero. That leaves us with

$$F = \sum_{j=k}^n a \frac{GmM_j}{R_j c^2} = m \left(\sum_{j=k}^n \frac{GM_j}{R_j c^2} \right) a \quad (4.10)$$

Here we are using m as gravitational mass. Inertial mass would appear to be

$$mi = m \left(\sum_{j=k}^n \frac{GM_j}{R_j c^2} \right) \quad (4.11)$$

The inverse proportionality to R rather than R^2 makes distant matter relatively much more important. Sciamia pointed out that in 1953 approximately 80% of the required mass was beyond the then observable universe. In Einstein's day use of such potential formulas was considered questionable because many people thought the observable universe might be infinite. Recently, as Ghosh points out [8], we can say that within the rather large uncertainty of the measurements, we have

$$\frac{mi}{m} = \sum_{j=k}^n \frac{GM_j}{R_j c^2} \approx 1 \quad (4.12)$$

Consider how equation (4.10) varies as we get close to one particular gravitating object M . The summation with respect to all other objects is still ≈ 1 . But the term for M grows larger and eventually must be considered separately. If we write "1" for the general summation, and add the term for a particularly nearby M , we have

$$\frac{mi}{m} = 1 + \frac{GM}{Rc^2} \quad (4.13)$$

Substituting (4.13) into $F=(mi)a$, we get exactly equation (4.8) which was derived only from equivalence and SRT without consideration of any general theory of inertia. Now let's take the reasoning backward. If we start with (4.13) and apply it to all the other masses in the universe, we will quickly see that we cannot sum the "1" every time. The result would get too large after only two masses were considered. So we conclude that the "1" must represent the sum of remaining masses. Summing (4.13) over all masses without the "1" we get (4.12). Substituting (4.12) into $F=(mi)a$ we get (4.10). Now add back the known Newtonian force of gravity, and eliminate all of the summation terms except those that pertain to one particular M , and we get the basic equation of inertia theory (4.9). We conclude, therefore, that the theory of inertia is consistent with and even derivable from the Principle of Equivalence.

Lorentz contraction appears to be a cause of equivalence, not an effect. In acceleration, a differential velocity caused by ever increasing contraction relative to an inertial frame causes inertial effects. In gravity, proximity is the cause. An inertial free-faller observes velocity-related Lorentz contraction of the objects it passes, but in orbital situations the direction will not even coincide with the gravity vector. The gravitational source is not clearly part of any of these reference frames as there is no source in an accelerated frame, so there is no basis for transforming the radius R by Γ . When inertia is developed from (4.10) forward, the Lorentz contraction explains inertia of moving objects, but real physical radius explains inertia of objects in a gravitational field. So Γ was omitted from (4.6).

V. BOUND AND UN-BOUND ACCELERATION

Does the double bending of light from speed gradient refraction provide an alternate explanation of light bending, or is it just a different way of expressing the curvature of space-time? This question may be more about philosophy than physics, unless different testable predictions can be made.

Petkov [8,9] recently pointed out that an accelerating spaceship paradox, similar to our elevator, has generated

conflicting opinions among physicists. If two spaceships with a thread between them accelerate identically, will the thread break? Or will the space between the ships contract? What we see in Figure 1, assuming one spaceship is at the top of the elevator and one at the bottom, is that to maintain a bound system with a fixed distance between them, the rear spaceship must accelerate more, and go faster, relative to any inertial frame. Each spaceship will have a different standard of length, but oddly, they may hold the same meter stick between themselves. Since the upper and lower light beams are not bound, the distance between them appears to increase in the elevator frame. If the spaceships accelerate identically in an inertial frame, they will drift apart in their own frames. Acceleration “fields” may overlap, yet be disjoint. The unity of an accelerated frame is determined by the binding of objects, not exclusively by a region of space. So *if* something causes curvature, it may be the combination of acceleration and binding.

Equation (4.9) implies that the extra inertia indicated in (4.8) is not universal, but is only felt when accelerating M and m with respect to each other. So M and m become “bound” and while their combined inertia does not increase, their inertia relative to each other does.

VI. CONCLUSION

There is a clear inference from equivalence that there is some type of inertial mass increase in a gravitational field. It is the purpose of the current paper to suggest that equivalence provides a more complete picture of gravitational effects than previously thought, correctly predicting full light bending, and that since the theory of inertia is derivable from equivalence, any theory based on equivalence must take account of it.

Einstein himself clearly was not satisfied with the status of inertia in GRT, as our quotes have shown. Many have tried to account for inertia and met with less than success, for example Davidson’s integration of Sciama’s inertia into GRT but only for a steady state cosmology [10], and the Machian gravity theory of Brans and Dicke [11]. Yet Mach’s idea hasn’t gone away, and now it seems that it cannot go away without also disposing of equivalence.

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